Suppression and enhancement of a drift-type instability in a magnetoplasma by a feedback technique

1971 J. Phys. A: Gen. Phys. 4 120

(http://iopscience.iop.org/0022-3689/4/1/016)

J. Phys. A: Gen. Phys., 1971, Vol. 4. Printed in Great Britain

Suppression and enhancement of a 'drift-type' instability in a magnetoplasma by a feedback technique

B. E. KEEN and R. V. ALDRIDGE†
UKAEA Research Group, Culham Laboratory, Abingdon, Berks., England
MS. received 1*st July* 1970

Abstract. This paper presents the experimental details and the results obtained when a feedback system is used to suppress or enhance drift instabilities present in a magnetoplasma. A nonlinear phenomenological theory is developed which predicts how the amplitude and frequency of the instability should change as a function of gain and phase shift in the feedback loop. Comparison of the results with this theory show good agreement. Further, the effect on the density and temperature profiles when the instability is suppressed has been investigated, and from these results a quantitative change in the cross-field diffusion constant has been inferred. Also, experiments are reported which show the effects of feeding back distinct azimuthal mode numbers $m = 0$, ± 1, and ± 2, as well as the results obtained using more than one separate system.

1. Introduction

In recent years, one of the problems in fusion research has been the possible correlation between the presence of finite amplitude micro-instabilities in a plasma, and the anomalously high diffusion of plasma across a containing magnetic field. In some degree, this has been substantiated by the experiments of Chu *et al.* (1967) and Hendel *et al.* (1968) who measured the density profile with and without a collisional drift instability present in their 'Q' machine plasma. From these results they calculated a change in the cross-field diffusion coefficient. Most containment devices in fusion physics are designed so that the effect of such instabilities on the cross-field diffusion is minimized. The usual method of achieving this is by using complex magnetic field geometries. However, another technique has been proposed by Artsimovich and Karteshev (1962) and Arsenin *et al.* (1969), in which a feedback method is used to stabilize the instability in the plasma. This method has been demonstrated (Arsenin *et al.* 1969) successfully on the flute instability, and has also been predicted (Arsenin and Chuyanov 1968) for the drift instability. This paper reports results obtained by a similar technique on a 'drift-like' instability. Preliminary reports of this work have been published by Keen and Aldridge (1969), and recent work on the same subject has been reported by Parker and Thomassen (1969) and by Simonen *et al.* (1969).

Any feedback method essentially relies on firstly being able to detect, unambiguously, the instability and then using this information to couple a signal back into the plasma. This feedback signal must be of the correct amplitude and phase such that a constraint is applied to the plasma, in order to damp out the instability. The experiments performed using this technique were of two types:

(i) Those which established the usefulness and the loop characteristics of the technique when suppressing a drift instability.

† Present address: Department of Mathematical and Physical Sciences, University of East Anglia, Norwich, England.

(ii) Those which studied the effect on the steady state plasma density and temperature, when suppressing or enhancing the instability.

In § 2, the recent linearized theories of feedback stabilization are briefly considered and a phenomenological nonlinear theory is developed. § 3 describes the apparatus and the techniques used to establish the nature of the instability. § 4 records the results obtained using the feedback in various azimuthal mode numbers and various configurations of suppressor plates. Further, this section presents the effect of suppression on the zero-order density and temperature profiles. § 5 compares the experimental results with the phenomenological theory developed in § 2, and continues by obtaining some quantitative measure of the change in the cross-field diffusion constant of the plasma. Finally, § 6 briefly discusses the future possibilities of the feedback technique.

2. Theory

Recently there have been a number of papers on the theory of the feedback technique and its application to the suppression of 'drift-type' instabilities in plasmas. Some of these are Arsenin and Chuyanov (1968), Simonen *et al.* (1969), Chen and Furth (1969), Furth and Rutherford (1969) and Taylor (1969).

Arsenin and Chuyanov (1968) considered the case in which a system of detectors and amplifiers could be used to excite, at a peripheral surface, a wave with a potential proportional to that of a collisionless drift instability present in the plasma. They suggested that by a suitable choice of phase and gain, that the phase velocity of the wave in the system formed by the plasma and electronic circuit could be higher than the Larmor drift velocity of the electrons. Then, by an exchange of energy between the wave and the resonant electrons, they predicted that this would lead to damping of the instability perturbations. This theory amounts to considering the change in the boundary conditions at the plasma surface when the feedback signal is present, and, therefore, it will be effective only on large-scale perturbations of the surface wave type. Small scale oscillations localized in the plasma will be unaffected by this technique, as they are insensitive to changes in the boundary conditions and will not be stabilized. Under the experimental conditions described here, this theory predicts unattainably large gain factors for suppression by a plate at the plasma boundary. In any case, the plates were situated not outside, but inserted into the plasma.

Simonen *et al.* (1969) have considered the effect of feedback on the collisional-type drift wave occurring in their 'Q' machine plasma. They approached the problem by considering the linearized two-fluid equations, in the 'slab' model approximation, and included a feedback source term $S = |\sigma|\tilde{n}_1 \exp(i\theta)$, proportional to the gain, phase change θ and the density perturbations $\tilde{n}_1$ in the plasma. They obtained a dispersion relation which predicted how the growth rate and frequency should behave in the presence of the feedback. Further, for an electron sink $|\sigma_e|$, they obtained stability criteria in the limiting cases as follows:

$$\frac{1}{\tau_\perp} > \frac{1}{\tau_\parallel} + |\sigma_e| \cos\theta \qquad (1)$$

for the case $\omega^* \gg 1/\tau_\parallel$ (long-wavelength limit).

$$\frac{2}{\tau_\parallel \tau_\perp} > \omega^*(2b\omega^* + |\sigma_e| \sin\theta) \qquad (2)$$

for the case $\omega^* \ll 1/\tau_{\parallel}$ (short-wavelength limit). Here $\tau_{\parallel}$ is the time for electrons to diffuse the parallel distance $1/k_{\parallel}$ ($k_{\parallel}$ is the parallel wavenumber), $\tau_{\perp}$ is the time for ions to diffuse the transverse distance $1/k_{\perp}$ ($k_{\perp}$ is the perpendicular wavenumber), ω^* is the drift frequency, and $b = k_{\perp}^2 r_L^2/2$, where r_L is the ion Larmor radius.

From equations (1) and (2) it can be seen that the optimum feedback phase for stabilization is 180° and 270° in the long- and short-wavelength limits, and there is an optimum gain (proportional to $|\sigma_e|$) for stabilization in each case. These authors have obtained fairly good agreement between these predictions and their experimental results. Similar criteria have been found independently by Furth and Rutherford (1969) and also by Chen and Furth (1969).

Taylor (1969) has approached the problem by considering the generalized case for any instability. He considers a detector which senses the potential ϕ at position x and in response to this signal charges up a suppressor element located at x'. A generalized response function of the feedback circuit is taken to be $g \exp(i\theta) G(x, x')$, where g is the gain, θ the phase angle, and $G(x, x')$ is real. The response of the plasma to an oscillating potential $\phi(x) \exp i\omega t$ is represented by a generalized conductivity tensor, $K_\omega(x', x)$ and if only potential oscillations are considered a dispersion equation is obtained:

$$\nabla \cdot \int dr' \epsilon_\omega(r, r') \nabla \phi(r') = 0 \tag{3}$$

where

$$\epsilon_\omega(r, r') = I + \frac{4\pi}{i\omega} K(r, r') \tag{4}$$

is the dielectric constant of the plasma. This dispersion equation determines the stability of the system through the eigenvalue ω.

When the suppressor is active, the oscillation frequencies are given by

$$\nabla \cdot \int dr' \epsilon_\omega(r, r') \nabla \phi(r') + g \exp i\theta \int dr' G(r, r')\, \phi(r') = 0. \tag{5}$$

Using perturbation theory (where g is small), the effect of the feedback suppressor on the real and imaginary parts of the eigenvalue $\omega = \omega_0 + i\gamma_0$ is found. Here ω_0 is the original instability frequency, and γ_0 is the corresponding growth rate. He finds that the new frequency ω and growth rate γ are given by

$$\omega = \omega_0 + gk \cos\theta \tag{6}$$

$$\gamma = \gamma_0 + gk \sin\theta \tag{7}$$

where

$$k = \frac{\int \phi^* G \phi}{\int \nabla \phi^* \left(\frac{\partial \epsilon}{\partial \omega}\right) \nabla \phi}. \tag{8}$$

Thus stabilization requires

$$gk \sin\theta > \gamma_0 \tag{9}$$

and so the effect of feedback may be to suppress or enhance instabilities according to the phase difference θ.

However, the above theories are all linearized theories, and do not allow for the positive feedback case when the instability is enhanced rather than suppressed. In order to explain this case a nonlinear theory must be used which limits the final signal level of the instability to a finite value. In the last few years there has been considerable interest in the nonlinear mechanisms which determine the saturation level of plasma instabilities. In fact, it has been shown that the Van der Pol (1922) type of nonlinear theory gives a good description of various kinds of nonlinear phenomena occurring in some plasma instabilities. These phenomena include mode locking and mode competition (Lashinsky 1965 a, 1965 b), periodic pulling (Abrams *et al.* 1969), frequency entrainment or 'synchronization' (Keen and Fletcher 1969) and 'asynchronous quenching' effects (Keen and Fletcher 1970). Further, it has been shown theoretically by Stix (1969), when considering finite-amplitude collisional drift wave oscillations, that a solution may be obtained 'which saturates in a manner similar to the Van der Pol solutions'. Consequently, as it has been shown that this type of differential equation gives a good description of finite amplitude collisional drift waves, the phenomenological approach has been adopted here in which the Van der Pol equation is taken to describe the density oscillations in the plasma. The equation in its simplest form without feedback is:

$$\frac{d^2 n_1}{dt^2} - (\alpha - 3\beta n_1^2)\frac{dn_1}{dt} + \omega_0^2 n_1 = 0 \tag{10}$$

where n_1 is the density perturbation, ω_0 the drift wave frequency, α is the linear growth rate ($\alpha/\omega_0 \ll 1$), and β is a nonlinear saturation coefficient which limits the final amplitude. This final amplitude a_0 without feedback is given by

$$a_0 = (4\alpha/3\beta)^{1/2}. \tag{11}$$

Now, consider a signal proportional to the density perturbations n_1 fed back into the system but altered in amplitude by an absolute gain g, and delayed in time by τ. Then this signal is represented by $gn_1(\tau)$, where the notation $n_1(\tau)$ is equivalent to $n_1(t-\tau)$. The quantity n_1 should be written $n_1(t)$ emphasizing that it is a function of time, whereas the term $n_1(\tau) = n_1(t-\tau)$ is a retarded one and relates to past time $(t-\tau)$, τ being a time lag. (In fact $\omega_0\tau = \phi$ the phase lag.) Then, with the feedback term included, the equation (10) becomes

$$\frac{d^2 n_1}{dt^2} - (\alpha - 3\beta n_1^2)\frac{dn_1}{dt} + \omega_0^2 n_1 + g\omega_0^2 n_1(\tau) = 0. \tag{12}$$

This equation is a simple example of a difference-differential equation (Minorsky 1962), and can be rearranged in the form

$$\frac{d^2 n_1}{dt^2} + \omega^2 n_1 = (\omega^2 - \omega_0^2) n_1 + (\alpha - 3\beta n_1^2)\frac{dn_1}{dt} - g\omega_0^2 n_1(\tau) = \sum H. \tag{13}$$

If a solution of the form $n_1 = a \sin \omega t$ is assumed, it can be shown that equation (4) can be brought into the form

$$\frac{d^2 n_1}{dt^2} + \omega^2 n_1 = F\{a(t), \omega\} \cos \omega t + f\{a(t), \omega\} \sin \omega t + \text{harmonics}. \tag{14}$$

If the calculation is limited to the fundamental frequency ω the solution in the first approximation is

$$a(t) = \frac{1}{2\omega}\int_0^t F\{a(\epsilon), \omega\}\, d\epsilon, \qquad 0 = \frac{1}{2\omega}\int_0^t f\{a(\epsilon), \omega\}\, d\epsilon. \tag{15}$$

For the transient condition the first part of equation (15) gives

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{1}{2\omega} F(a, \omega) \tag{16}$$

and for the stationary state one has

$$F(a, \omega) = 0, \qquad f(a, \omega) = 0. \tag{17}$$

Therefore, if one calculates the coefficients of cos ωt and sin ωt from the expression ΣH, in equation (13), the following conditions are obtained for the stationary state:

$$a_0^2 - a^2 = -g\left(\frac{\omega_0^2}{\alpha\omega}\right) a_0^2 \sin\phi \tag{18}$$

$$\omega^2 = \omega_0^2(1 + g\cos\phi). \tag{19}$$

Equation (18) shows that as the gain g is increased from $g = 0$ the amplitude a will increase or decrease according to the sign of sin ϕ. Optimum suppression is achieved with

$$\sin\phi = -1 \qquad (\text{i.e. } \phi = -90^\circ \text{ or } 270^\circ) \tag{20}$$

and, suppression occurs when $g = \alpha/\omega_0$ (since $\omega = \omega_0$ at $\phi = -90^\circ$).

The predictions of this phenomenological theory are compared with the experimental facts in § 5. It is seen that the predictions for the frequency shift ($\Delta\omega$) and the change in amplitude squared $\Delta(a_0^2 - a^2)$ (proportional to $\Delta(\alpha_0 - \alpha)$ or $\Delta(\gamma_0 - \gamma)$) in terms of the gain g and phase shift ϕ or θ are very similar to those in Taylor's (1969) generalized theory.

3. Experimental details

3.1. *Hollow cathode arc apparatus*

The plasma employed was a hollow cathode arc discharge (Woo and Rose 1967) running in argon. The cross section of the apparatus is shown in figure 1. This can be divided into three main sections: (i) a production region, (ii) a baffle region, and (iii) a $1\frac{1}{2}$ metre experimental region. The plasma was produced at a hollow tantalum cathode (0·3 cm inside diameter), through the centre of which the neutral argon gas entered (flow rate $\sim$1·0 atm cm^3 s^{-1}). Two copper baffles 4 cm in diameter, 10 cm long and 15 cm apart formed the intermediate baffle region. These baffles or limiters restricted the plasma column to about 4–5 cm diameter inside the 10 cm diameter glass tube of the drift (experimental) region. The plasma was terminated at a copper plate anode ($\sim$4 cm diameter) which also served as an auxiliary gas feed (flow rate $\sim$0·3 atm cm^3 s^{-1}). Each region was differentially pumped by one 15 cm diameter diffusion pump (speed 1400 litres/s each). The axial magnetic field in the experimental region was separately variable from that in the production and baffle region and was uniform to better than 0·5% throughout each region, when applied separately. The current through the arc was kept constant at 20 A throughout these experiments.

It has been shown (Woo and Rose 1967) that a relatively quiescent plasma can be obtained if the arc running conditions are arranged such that $\Omega_i\tau_i \ll 1$ in the cathode region, $\Omega_i\tau_i \leqslant 1$ in the baffle region and $\Omega_i\tau_i \geqslant 1$ in the experimental region. (Here $\Omega_i = eB/M_ic$ is the ion cyclotron frequency, and τ_i is the mean ion collision frequency.) These inequalities may be satisfied by varying the axial magnetic field or alternatively, by varying the gas flow rates or pumping speeds to the various regions. Then the experimental region is well isolated from the production region where conditions are ideal for the formation of many instabilities.

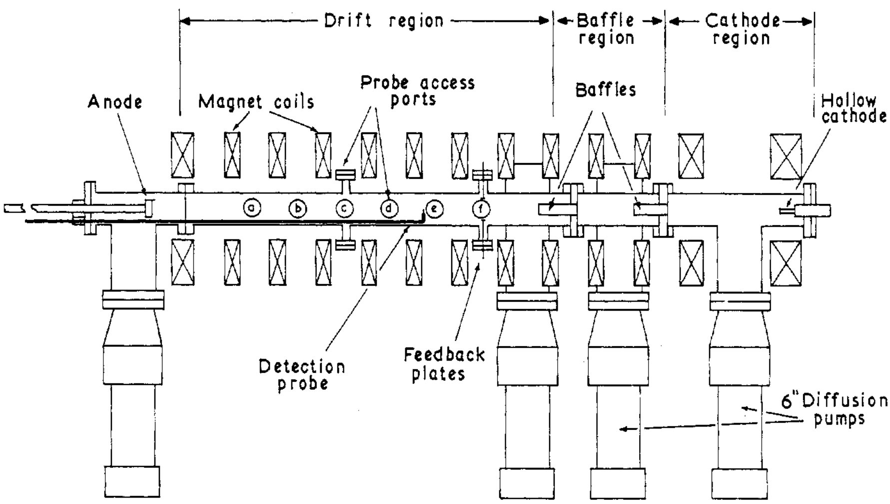


Figure 1. A cross section of the hollow-cathode arc apparatus.

However, in this experiment the flow rates were kept constant at the values mentioned above, and the magnetic field in the production and baffle region was kept constant at 0·5 kG and in the experimental region was varied between 0·5 and 1·0 kG. Under these conditions $\alpha = \Omega_i \tau_i \leqslant 1$ in the experimental region and, consequently an instability was present.

3.2. *Diagnostic methods*

Between each magnetic field coil were spaced a set of ports (totalling 6 sets shown marked as (*a*)–(*f*) on figure 1) along the tube in the experimental region. Each set comprised four ports arranged symmetrically around the tube at 90° intervals, except for one set which was rotated 60° (at 60°, 150°, 240° and 330°). Interchangeable probes could then be inserted at any of these positions. The single probes could be used in their floating or ion-based configurations and could be moved radially across the plasma.

Another single probe could be moved axially along 1 m of the column, and also, could be rotated to any radial position. This is marked in figure 1. Density and temperature profiles were obtained using a radially moveable double probe (Johnson and Malter 1950). The probe was constructed from two tungsten wires 0·05 cm in diameter and separated 0·10 cm apart which protruded 0·10 cm from a boron nitride holder 0·30 cm in diameter. The probe moved on a micrometer carriage to the centre of the column. The ion temperature T_i was estimated from the broadening of emitted spectral lines, but owing to experimental error it was only possible to obtain an upper limit of $T_i < 0·5$ eV.

In order to identify the instability unambiguously, it was also necessary to know the radial electric field E_r and the zero-order rotation frequency ω_r of the column. These measurements were discussed by Aldridge and Keen (1970), and the methods were explained in more detail at that time. Briefly, the radial electric field E_r was obtained by three different methods, and was deduced from:

(i) the corrected spatial variation of the floating potential ϕ_f.

(ii) the spatial variation of plasma potential ϕ_p as measured using a thermionically emitting probe (Kemp and Sellen 1966).

(iii) the spatial variation of the plasma potential ϕ_p as deduced from the 'knee' in the ln i_e against V_a plot of a single probe curve. (Here V_a is the applied potential to the probe, and i_e is the resulting electron current drawn by the probe.)

The rotation of the arc column as a function of radius was determined from measurements on a single-sided ion saturated probe (Brundin 1964). A single probe shielded on one side by boron nitride was biased to ion saturation current. The probe was faced away from the rotation, the ion-saturation current i_s measured, and then faced into the flow and the difference Δi in the saturation current measured. The difference was related to the rotational velocity v_0 through the relationships $v_0 = (\Delta i/i_s)(T_e/M_i)^{1/2}$. Hence, in this way the rotational frequency $\omega_r = v_0/r$ was obtained as a function of radius r.

3.3. *Feedback system*

The electronic apparatus of the feedback system is shown schematically in figure 2. A signal from an ion-biased probe 1 (see also figure 2(*b*)) proportional to

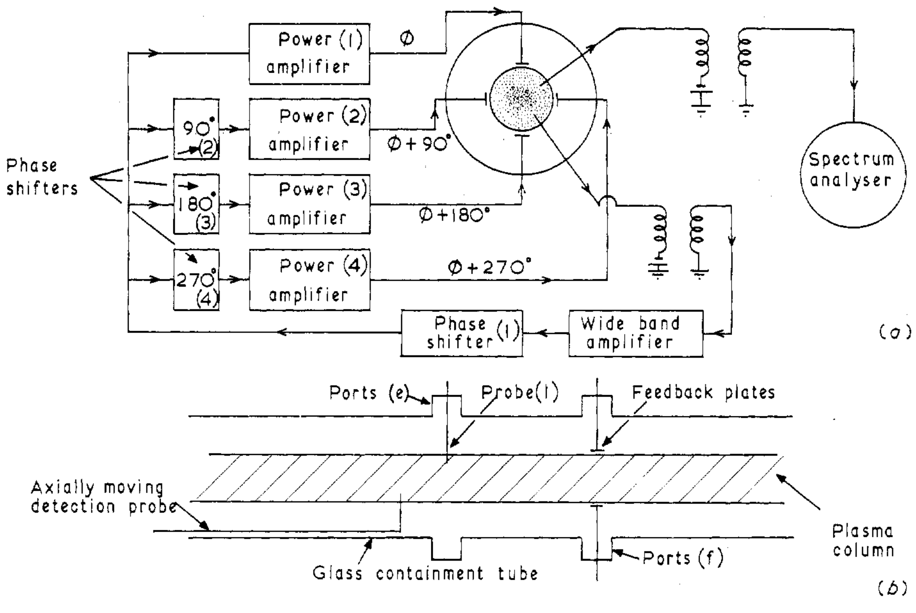


Figure 2. (*a*) Electronic set up; (*b*) Cross section of plasma showing relative positions of different probes.

the density perturbations in the plasma was coupled to a wideband amplifier with a gain capability of −20 dB to 40 dB and a bandwidth of 1·5 Hz to 400 kHz. From the amplifier the signal was passed through a unity gain phase shifter 1 which was capable of shifting the phase up to 450°. The system was then coupled through similar phase shifters 2, 3 and 4 and power amplifiers 1, 2, 3 and 4 to four plates spaced symmetrically around the plasma at 90° intervals. The power amplifiers were capable of delivering up to 25 W each within a bandwidth 0–250 kHz. The system of phase shifters and the four plates make it possible to feedback particular azimuthal mode numbers $m = 0, 1, -1, \pm 2$, etc., into the plasma, or, alternatively, one, two, three or four separate plate systems could be used. The axially moving probe was used to detect the effect of feedback on the instability, since this could be moved radially and axially in the plasma to observe that the effect was consistent throughout the whole volume. The output of this probe was displayed on a spectrum analyser from which measurements of relative amplitude and frequencies could be obtained.

The plates were normally rectangular in shape, 1·2 cm long (parallel to the axial direction) and 5 mm wide (in the azimuthal direction). They could be moved radially, and were normally kept at ports (*f*) (figures 1 and 2(*b*)). The relative positions of the input probe 1 (at port (*e*)), the plates (port (*f*)), and the axially moveable probe are shown in figure 2(*b*).

Finally, by interposing a tone-burst generator between phase shifter 1 and the power amplifiers, it was possible to obtain information about the growth-time of the instability. The tone-burst generators acted as a periodic fast gate on the feedback signal, and so the instability in the plasma was suppressed and then allowed to return, as the signal was 'on' and then 'cut off'. The instability signal was monitored on an oscilloscope and photographed so that the decay and rise time could be obtained from the trace.

4. Results

4.1. *Instability measurements*

For slight variations of the external conditions (e.g. arc potential at the constant current of 20 A, magnetic field, etc.) the fundamental frequency of the instability was found to lie in the range 5·5–8·0 kHz. The instability at the lower frequency had a much narrower halfwidth ($\sim$0·4 kHz), whereas, nearer 8·0 kHz, the halfwidth was approximately 1 kHz. Figures 3(*a*) and 3(*b*) show the amplitude and phase of the instability as a function of radius taken at 5·5 kHz frequency ($B_0 = 0{\cdot}5$ kG). The potential oscillations $\tilde{\phi}_1$ of the instability were observed from a floating probe, and it is seen that a π change of phase occurs at the centre of the column where the amplitude is a minimum. The phase appears to change slightly as the probe is moved further out, but probably this is related to the rotation of the plasma. Density perturbations $\tilde{n}_1$ were checked as a function of radius and the oscillations were found to be approximately electrostatic (i.e. $\tilde{n}_1/n_0 \simeq e\tilde{\phi}_1/kT_e \simeq 15\%$). Four probes placed on equal radii around the plasma showed that the fundamental frequency ω_0 of the instability had an $m = 1$ azimuthal mode number. Also present was some component at $2\omega_0$ with $m = 2$ with an amplitude of approximately 25% of the fundamental, and some at $3\omega_0$ ($m = 3$) with less than 10% of the fundamental amplitude. From axial amplitude and phase measurements the fundamental was found to have a wavelength λ_z much greater than 200 cm.

The density profile of the plasma taken under these same conditions is shown in figure 4(*a*). Over a certain range of radii (0·6–2·5 cm) the density changes exponentially with radius with a constant inverse scale length

$$\kappa = \frac{1}{n_0}\left(\frac{\partial n_0}{\partial r}\right) = 0{\cdot}65 \pm 0{\cdot}05\ \text{cm}^{-1}.$$

The corresponding temperature profile is shown in figure 4(*b*). The rotation frequency ω_r, measured with the one-sided probe, was approximately constant at

$\omega_r = 3{\cdot}5 \pm 0{\cdot}6$ kHz in the range of radius from 0·5 to 2·1 cm. The average value of the radial electric field E_r was found to be $-1{\cdot}0 \pm 0{\cdot}4$ V cm^{-1}.

In view of these results the instability was identified as a rotationally convected collisional drift instability (Aldridge and Keen 1970). Summarizing, the instability was predominantly an $m = +1$ azimuthal mode, with a frequency ω in the range

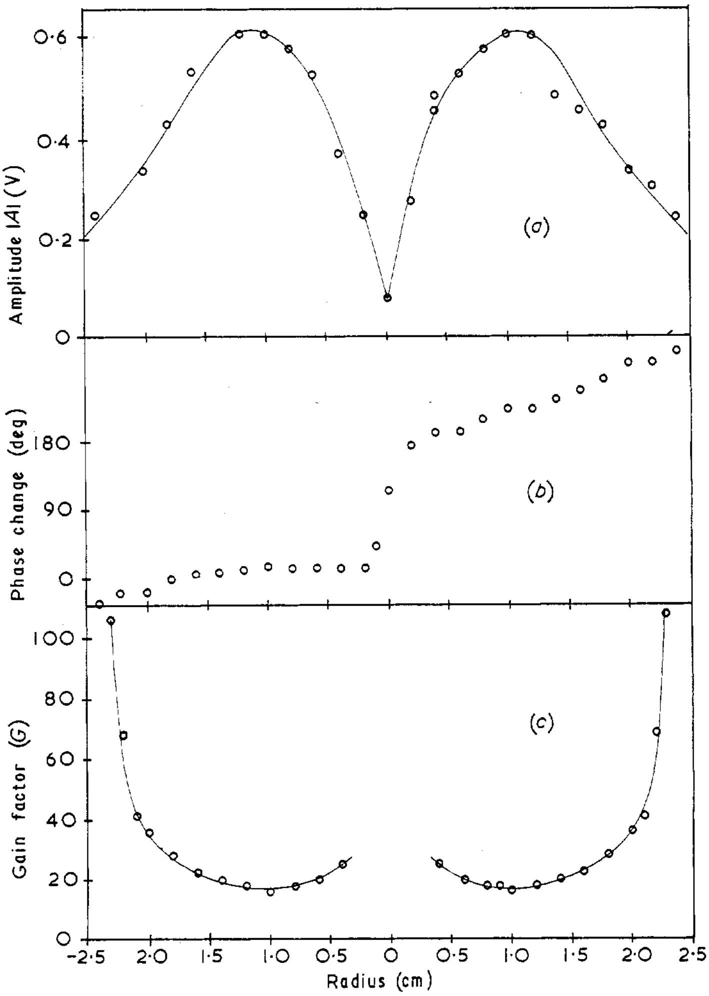


Figure 3. (*a*) Amplitude of potential oscillations as a function of radius *r*. (*b*) Relative phase of oscillations as a function of radius; (*c*) Oscillator gain *G* required for suppression for one plate at different radii *r*.

5·5–8·0 kHz (i.e. $\omega \ll \Omega_i$) depending upon the magnetic field B_0 between 0·5 and 1·0 kG. This compares with a theoretically predicted value of 5·0 to 7·0 kHz for the same magnetic field range. The axial wavenumber was found to be $k_z \ll 0{\cdot}03$ cm^{-1}, in comparison with a theoretically predicted maximum growth rate for $k_z \to 0$. Also, experimentally it was found that $\alpha = \Omega_i \tau_i \lesssim 1$ which is a necessary condition for the instability to appear. Therefore, it was inferred that this predicted 'drift-type' oscillation was the one present in the plasma.

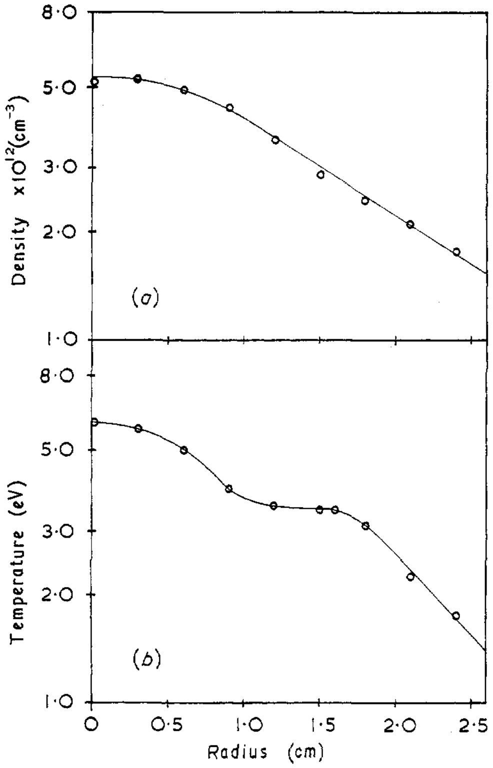


Figure 4. Shows (*a*) the density and (*b*) the temperature plotted against radial position r.

4.2. *Feedback results*

4.2.1. *Single plate system.* Initial experiments on the feedback system were performed using just one plate as the suppressor element. The power amplifiers 2, 3 and 4 were disconnected from their corresponding plates. The sensing probe (1) was arranged so that it was in the same axial plane and the same relative azimuthal position as the suppressor plate, in order that there was no relative azimuthal phase difference between the probe and plate. Therefore, the only phase change would be that in the electronic system. Initially, the gain in the system was set at an arbitrary value and the phase angle ϕ was varied until an optimum decrease in amplitude was found. Once this phase angle was set, the gain value G on the wideband amplifier was changed until suppression was achieved. The effect of the feedback is shown in figure 5. Figure 5(*a*) shows a spectrum analysis near ω_0 without feedback to the plasma; figure 5(*b*) illustrates the effect for optimum suppression of the instability; and figure 5(*c*) shows the enhancement achieved when inserting a phase shift of 180° in the feedback loop after conditions had been set for optimum suppression. Under these conditions the gain factor G for optimum suppression was measured as a function of radius r, at which the plate suppressor was set. This is shown plotted in figure 3(*c*). It is seen that the minimum gain ($G \simeq 17$) is required at about $r = 1{\cdot}0 \pm 0{\cdot}1$ cm, which corresponds closely to the maximum in the instability amplitude. This condition was found, by Simonen *et al.* (1969). Near the outside

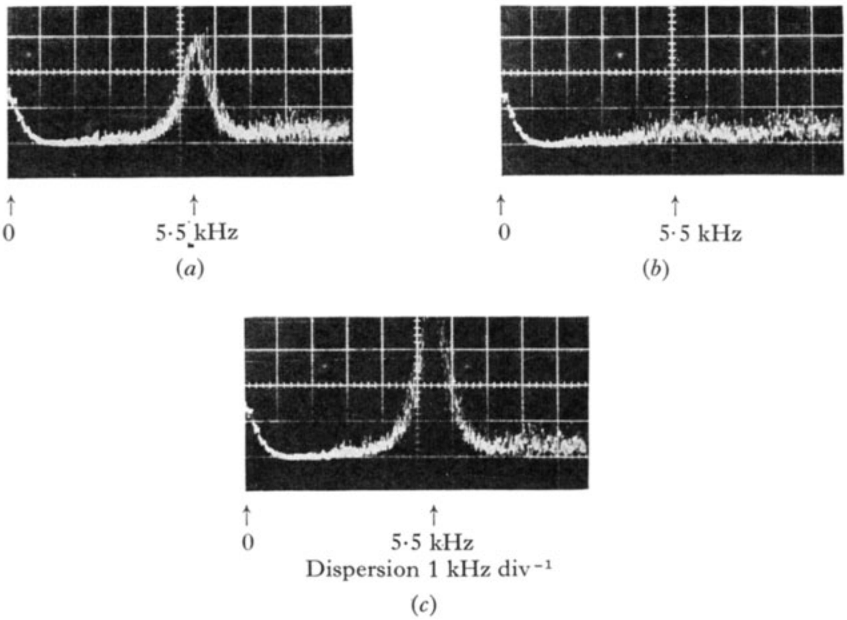


Figure 5. Spectrum analyser output showing the instability at 5·5 kHz for (a) no feedback, (b) optimum feedback for suppression, and (c) as for (b) but with the phase angle changed by 180° to cause enhancement.

of the column ($r \simeq 2{\cdot}5$ cm) gain factors in excess of 500 would not suppress the instability. Throughout the remaining experiments the plates were kept in the range of radii 1·6–2·2 cm in order that they did not interfere with the plasma conditions, such as the density profile, radial electric field, electron temperature, etc.

A study of the effect of gain and phase shift in the feedback loop on the instability amplitude a was made. Figure 6 shows the reduced amplitude a/a_0 plotted against

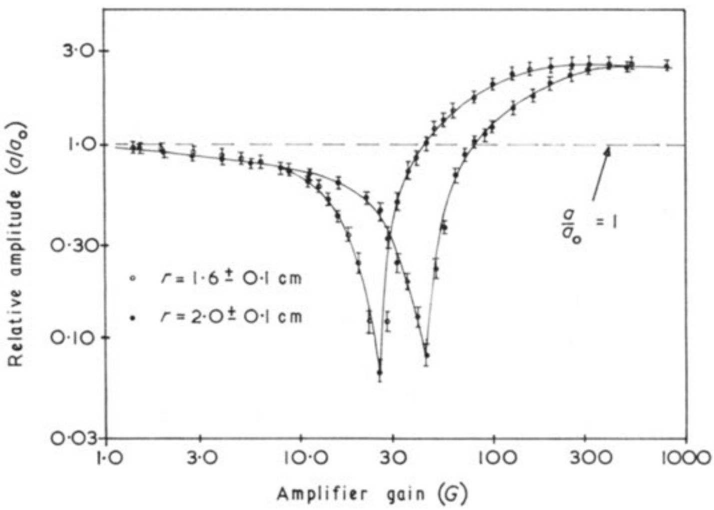


Figure 6. Reduced amplitude a/a_0 against amplifier gain for the radii $r = 1{\cdot}6 \pm 0{\cdot}1$ cm and $r = 2{\cdot}0 \pm 0{\cdot}1$ cm with phase angle ϕ set for optimum suppression.

the gain G in the wideband amplifier, for the plate set at two different radii $r = 1\cdot6 \pm 0\cdot1$ cm and $r = 2\cdot0 \pm 0\cdot1$ cm. These curves were taken with the phase angle ϕ set for optimum suppression. It is seen that, at this phase setting, below a certain gain the feedback has a stabilizing effect, while above it the system becomes progressively more unstable. It was checked that the instability level was consistent throughout the plasma, at the gain setting for minimum amplitude, by moving the axial probe both radially and longitudinally along the column.

The effect of varying the phase angle ϕ is shown in figure 7 for the cases (*a*) $G = 25\cdot2$ and (*b*) $G = 12\cdot6$. These were taken when the plate was set at a radius $r = 1\cdot8 \pm 0\cdot1$ cm. The gain value $G = 25\cdot2$ corresponds to the optimum value for suppression at this value of radius. It is seen that there is a minimum value at a phase angle of 270°, while there is a maximum and a corresponding enhancement at $\phi = 90°$.

At this stage, an experiment was tried in which the suppressor plate was increased in size to a circular plate of 2·0 cm diameter. This proved to be less successful in suppressing the instability and was probably due to imposing a definite phase angle ϕ on the whole area of the plate (which subtended an azimuthal angle of 50° to the centre of the column). However, the required phase angle for suppression is not constant over this azimuthal extent of the plasma instability, and thus only partial suppression occurred. Consequently, experiments were continued with the original plates which appeared to operate effectively.

4.2.2. *Multi-plate system.* Experiments were continued with the set-up as shown in figure 2(*a*), that is using the four phase shifters, power amplifiers and plates. By correctly setting the phase on the phase shifters 2, 3 and 4, definite azimuthal modes,

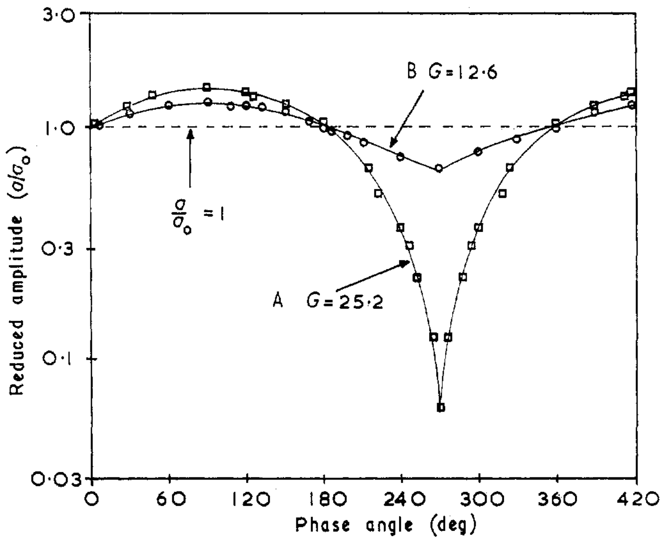


Figure 7. Reduced amplitude a/a_0 against phase angle ϕ for A, $G = 25.2$ and B, $G = 12\cdot6$.

numbers 0, -1, 1, ± 2, could be selected and fed back with a total phase shift determined by phase shifter 1. The results of using this technique are shown in figure 8. Figure 8(*a*) shows the spectrum analysis of the instability without feedback, and this indicates the relative amplitude occurring at the second and third harmonics compared with the fundamental at 5·5 kHz. Figure 8(*b*) shows the effects of feedback when an $m = 0$ mode is applied and it is seen that very little difference is found. For the $m = 1$ mode, figure 8(*c*) shows that the fundamental at 5·5 kHz is almost completely suppressed under optimum conditions, whereas the second harmonic (11 kHz, $m = 2$) is slightly enhanced in amplitude. Figure 8(*d*) shows that for an $m = 2$ feedback signal, the oscillations at 5·5 kHz are completely unaffected, whereas the 11 kHz signal is reduced in amplitude. Finally, figure 8(*e*) shows the feedback applied to only one plate, and here suppression effects are observed for all frequencies and modes, presumably because one plate can feedback a mixture of modes simultaneously.

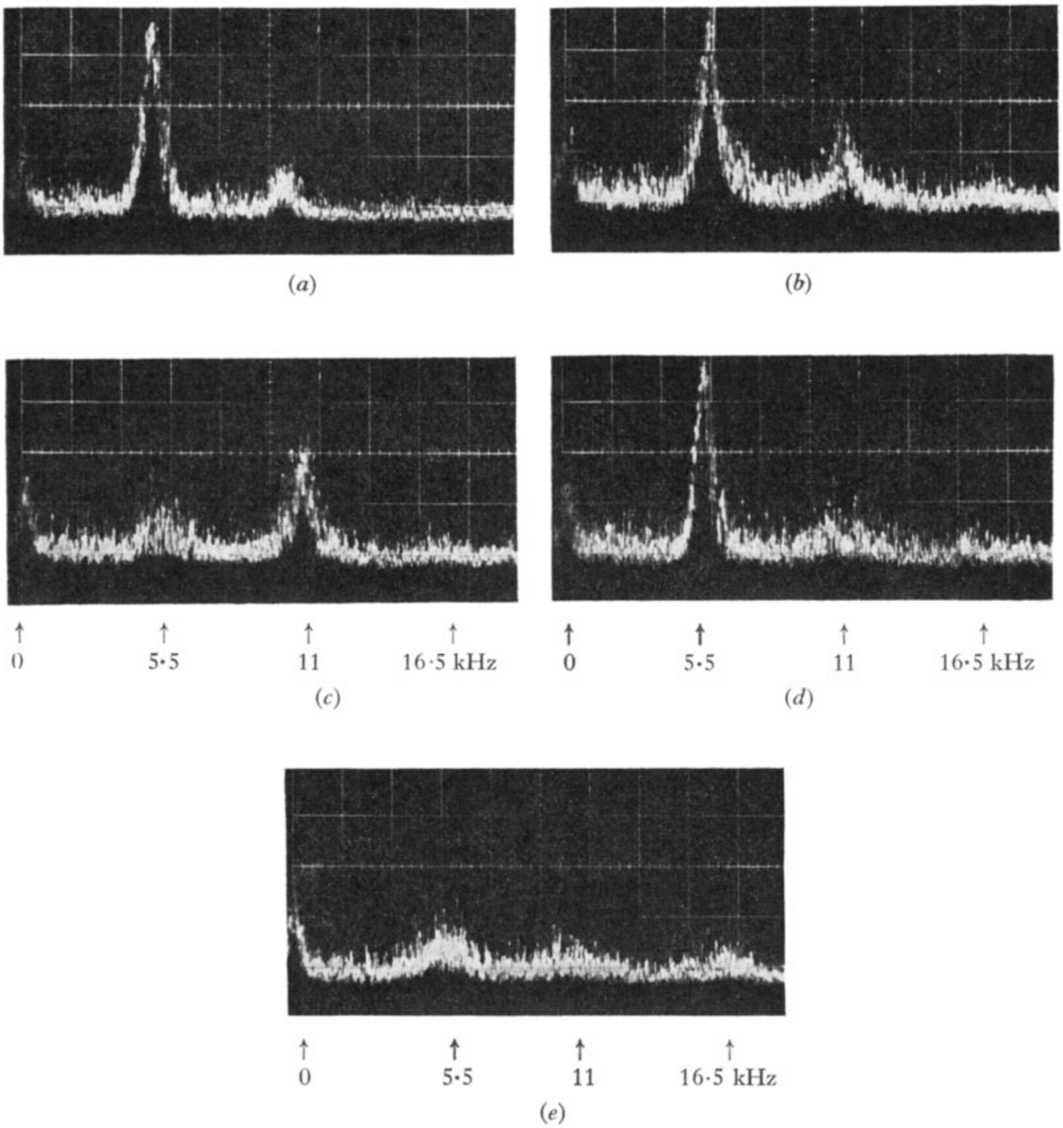
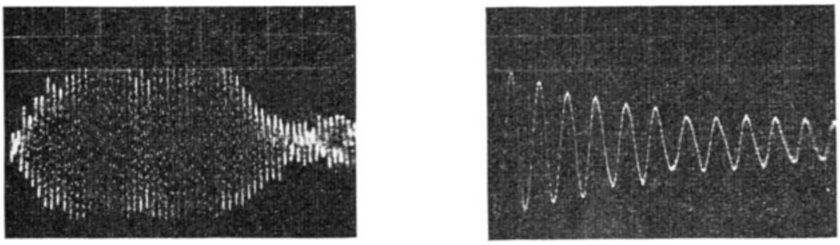


Figure 8. Spectrum analysis of the instability showing fundamental (5·5 kHz), second harmonic (11·0 kHz) and third harmonic (16·5 kHz) for the cases (*a*) with *no* feedback, (*b*) feedback in $m = 0$ mode, (*c*) feedback in $m = +1$ mode, (*d*) feedback $m = |2|$ mode, and (*e*) feedback in a mixture of modes on one plate.

When the instability occurred at the higher frequency ($\sim 7{\cdot}0$ kHz) the spectrum analysis showed the signal to be rather broad with a halfwidth greater than 1·0 kHz. When it was tried to suppress this instability using one plate, complete suppression was not achieved. Consequently, two independent feedback systems were employed. These consisted of two separate pick-up probes (ion-based), two wideband amplifiers, two phase shifters feeding two separate plates, via two power amplifiers. By successively changing the gain and phase of each system it was possible to suppress the broader instability. The effects obtained on the density and temperature profiles under these conditions will be mentioned in § 4.

Finally, as mentioned in § 3, the growth rate of the instability was measured by periodically gating the feedback signal with a tone-burst generator. Figure 9(*a*) shows a typical photograph of the burst of instability signal when the feedback signal is



Feedback off Feedback on
Scale: 1 ms div^{-1}
(*a*)

Feedback on
Scale: 0·2 ms div^{-1}
(*b*)

Figure 9. (*a*) Shows the burst of the instability obtained when the feedback signal is gated off for 32 cycles, and (*b*) shows expanded scale of the decay of the instability.

gated-off for a period and it is seen that there is a certain rise time and decay time associated with the leading and trailing edges of the burst of signal. Figure 9(*b*) shows the trailing edge on an expanded scale. These photographs were analysed and average values for a growth rate from the rising signal α_r and from the decaying signal α_d were obtained, as shown in figure 10. These results were obtained with the instability at 5·5 kHz. The parameter α is obtained by considering the transient solution of the undisturbed Van der Pol equation (equation (10)).

In this case the return to equilibrium is given by

$$a(t) = a_0[1+\{(a_0/a_1)^2-1\}\exp(-\alpha t)]^{-1/2} \tag{21}$$

where $a(t)$ is the instantaneous value of the amplitude at time t and a_1 is the initial value at time $t = 0$. It is seen that the initial growth rate at $\tau = 0$, depends upon the initial amplitude value a at $t = 0$. Therefore in order to obtain a value for α it was necessary to fit the curve given by equation (21) to the data.

4.2.3. *Results on the dc properties of the plasma.* Once the conditions for suppression of the instability had been established, and their characteristics had been studied, the effect on the density and temperature profiles with and without the instability

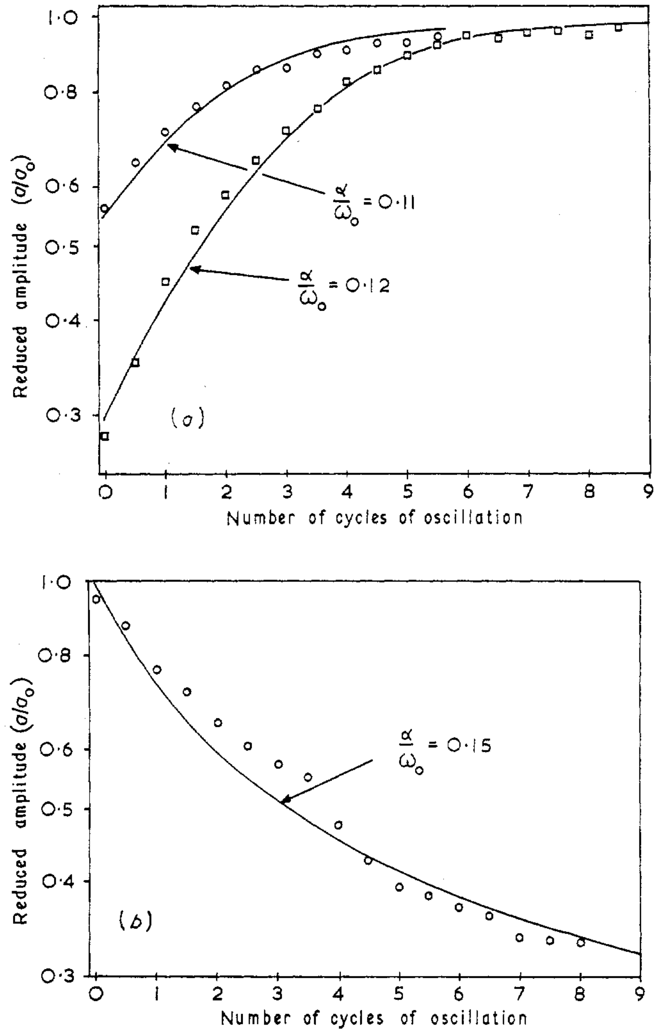


Figure 10. The logarithm of the reduced amplitude a/a_0 plotted against the number of periods of the signal for (*a*) the rise time, and (*b*) the decay time of the instability.

present were investigated. This was carried out using the radially moveable probe with the instability present and then with it 'switched off', at each radial position in the plasma. The results are shown in figures 11(*a*) and 11(*b*) for a single plate, single feedback loop system acting on the instability at 5·5 kHz. It was seen that the main effects of removing the instability were that the density profile was 'sharpened up', and that an inhomogeneity in the temperature profile was smoothed out. The radial position of this inhomogeneity corresponded with the maximum in the instability amplitude. When the instability was stabilized, it is seen that the density was higher at the centre and fell off faster at the periphery of the column, in comparison with when the oscillation was present. This is a consequence of the fact that the arc current was stabilized and thus the current $I = e\int_0^{r_0} 2\pi n(r)\, V_{\parallel}(r)\, r\, dr$ remained constant. Here e is the electronic charge, $n(r)$ is the density and $V_{\parallel}(r)$ the parallel electron velocity at a particular radius r of the column. If $V_{\parallel}(r)$ falls in a monotonic manner, any decrease in the cross-field diffusion constant $D_{\perp}$ would manifest itself by increasing the density at the centre and decreasing it at the outside of the column. This is observed experimentally when the instability is suppressed, and so it is inferred that suppression reduces the cross-field diffusion rate of the plasma.

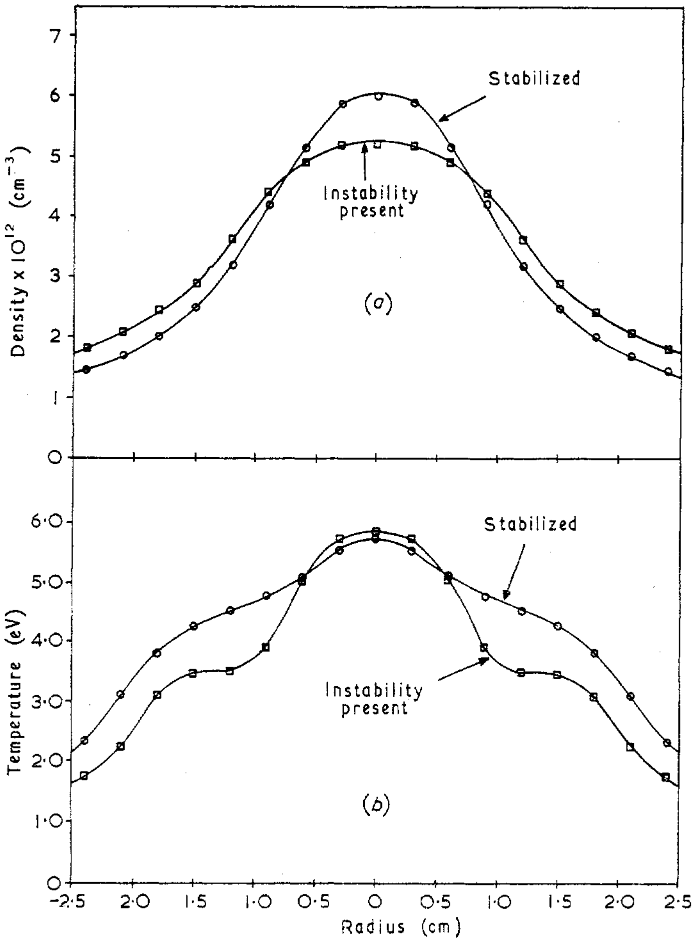


Figure 11. The effect of the removal of the instability at 5·5 kHz on (*a*) the density profile, and (*b*) the temperature profile.

The effect of one or more systems on the density profile of the plasma, when the broad instability at approximately 7 kHz is suppressed, is shown in figure 12. The reduced density profile $n(r)/n_I(0)$ with the instability present is shown in figure 12(*a*). Here $n_I(0)$ represents the density at the centre ($r = 0$) when the instability is present, and $n(r)$ is the density at any radial position r. The effect of one system is shown in figure 12(*b*), and it is seen that the central density rises by about 9% which is less than the central density enhancement ($\sim$ 16%) achieved on the narrower instability at 5·5 kHz using one plate suppressor. Two independent systems were then employed and the effect is shown in figure 12(*c*); here the central density is enhanced by approximately 18%. Therefore under certain conditions it appears that multi-plate, multi-independent systems might give an improvement over one system. The equipment

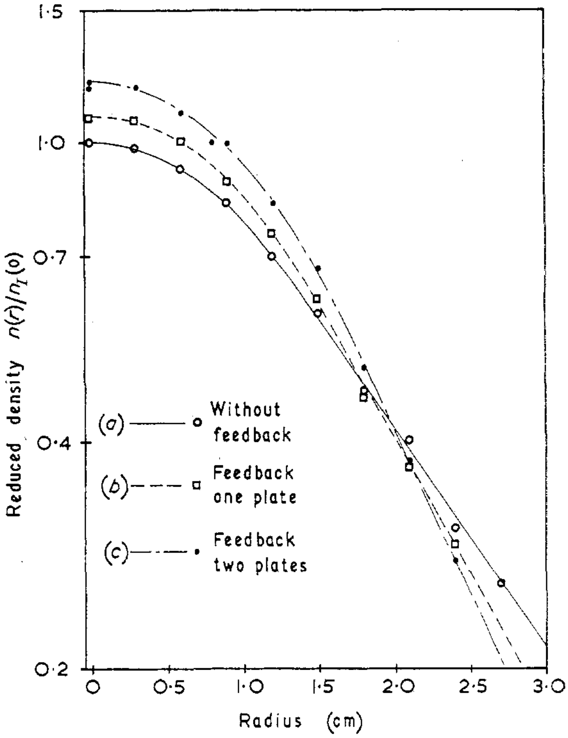


Figure 12. Shows the reduced density profiles $n(r)/n_l(0)$ obtained when (*a*) the instability at approximately 7 kHz is present, (*b*) when it is suppressed using one plate and one feedback loop, and (*c*) when it is suppressed using two plates and two feedback loops. Here $n_l(0)$ is the density at the centre with the instability present.

for more than two systems was not available and so could not be attempted, although this could be an interesting experiment.

5. Interpretation of results

5.1. *Feedback system*

A comparison is made here between the results obtained on the single plate, single feedback loop system and the theoretical predictions made by the phenomenological theory developed in § 2. In that section, equation (18) predicted that the square of the signal level a^2 should fall linearly as a function of the absolute gain g (or relative gain G) in the feedback loop, when the phase is kept at a constant value. The results shown on figure 6 have been replotted in figure 13 as the square of the reduced amplitude $(a/a_0)^2$ against increasing amplifier gain G. This is shown for the suppressor plate set at the two different radii, $r = 1{\cdot}6 \pm 0{\cdot}1$ cm and $r = 2{\cdot}0 \pm 0{\cdot}1$ cm. It is seen that a good linear relationship is obtained experimentally.

In figure 7, a minimum is obtained in reduced amplitude a/a_0 at a phase angle of approximately 270° and corresponding maximum occurs at approximately 90°. This is as predicted by the theory (equations (18) and (21)). Further equation (18) predicts that as the phase angle ϕ is varied at constant gain g (or G), the square of

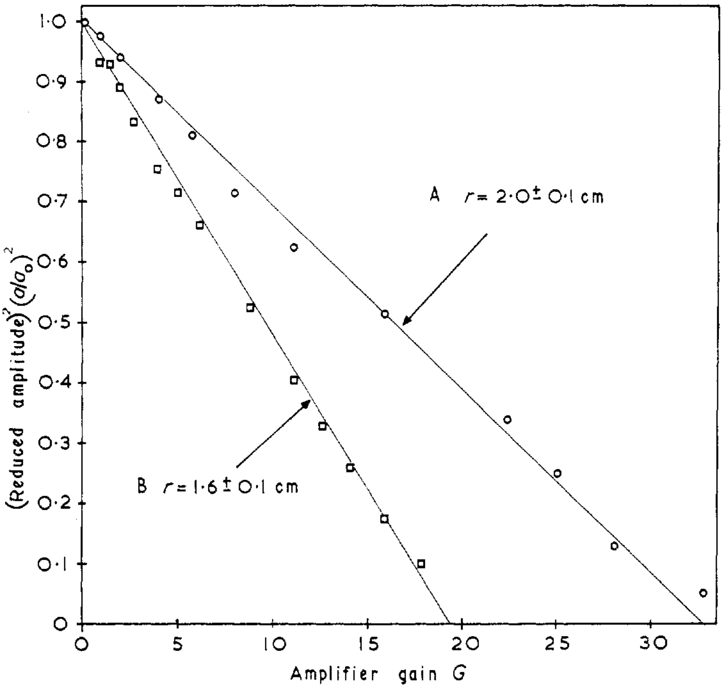


Figure 13. The square of the reduced amplitude $(a/a_0)^2$ plotted against amplifier gain G for the conditions when the suppressor plate is set at a radius r such that A, $r = 2{\cdot}0 \pm 0{\cdot}1$ cm and B, $r = 1{\cdot}6 \pm 0{\cdot}1$ cm.

the amplitude a^2 should vary proportional to sin ϕ. Figure 14 shows the square of the reduced amplitude $(a/a_0)^2$ plotted against (*a*), the phase angle ϕ and (*b*), sin ϕ. It is seen that the variation is as predicted (within experimental error) for the three gain values $G = 25{\cdot}2$, $12{\cdot}6$ and $7{\cdot}9$.

A calibration for the absolute gain g in terms of the relative gain G of the system was achieved with the aid of equations (19) and (20). Since, for optimum suppression ($G = 25{\cdot}2$) equation (20) gives $g = \alpha/\omega_0 (\ll 1)$, and then equation (19) gives $2\Delta\omega/\omega_0 = g \cos\phi$ (where $\Delta\omega = \omega - \omega_0$). The frequency shift $\Delta\omega$ was measured as a function of $\cos\phi$, and this is shown plotted in figure 15. A reasonable linear relationship is obtained as predicted, and the slope of the line is proportional to $g = \alpha/\omega_0$ $(= 0{\cdot}12 \pm 0{\cdot}02)$. This allows the absolute gain to be calibrated in terms of the amplifier gain $G(g = \gamma G)$ and so the constant $(\gamma = 4{\cdot}5 \times 10^{-2})$ is obtained. As a consequence, the theoretical expression of $(a/a_0)^2$ as a function of phase angle ϕ for each gain value, was calculated absolutely using equation (18). These resulting variations are shown as the continuous lines in figure 14.

This value of $\alpha = (0{\cdot}12 \pm 0{\cdot}02)\omega_0$ can be compared with the directly measured linear growth rate values obtained from the rise times and decay times of the instability, as mentioned in § 4. These values were $\alpha_r = (0{\cdot}12 \pm 0{\cdot}02)\omega_0$ and $\alpha_d = (0{\cdot}15 \pm 0{\cdot}03)\omega_0$. It is seen that the values obtained by the direct and indirect methods show good agreement.

It is seen that by adopting the phenomenological approach to this problem, in which a nonlinear equation of the Van der Pol type is used to explain the amplitude saturation conditions of this instability, relationships can be obtained between the amplitude a and frequency shift $\Delta\omega$ as a function of gain g and phase shift ϕ in the feedback loop. Comparison with measurements show the predicted variations and a consistent value for the growth rate α is obtained within the experimental error.

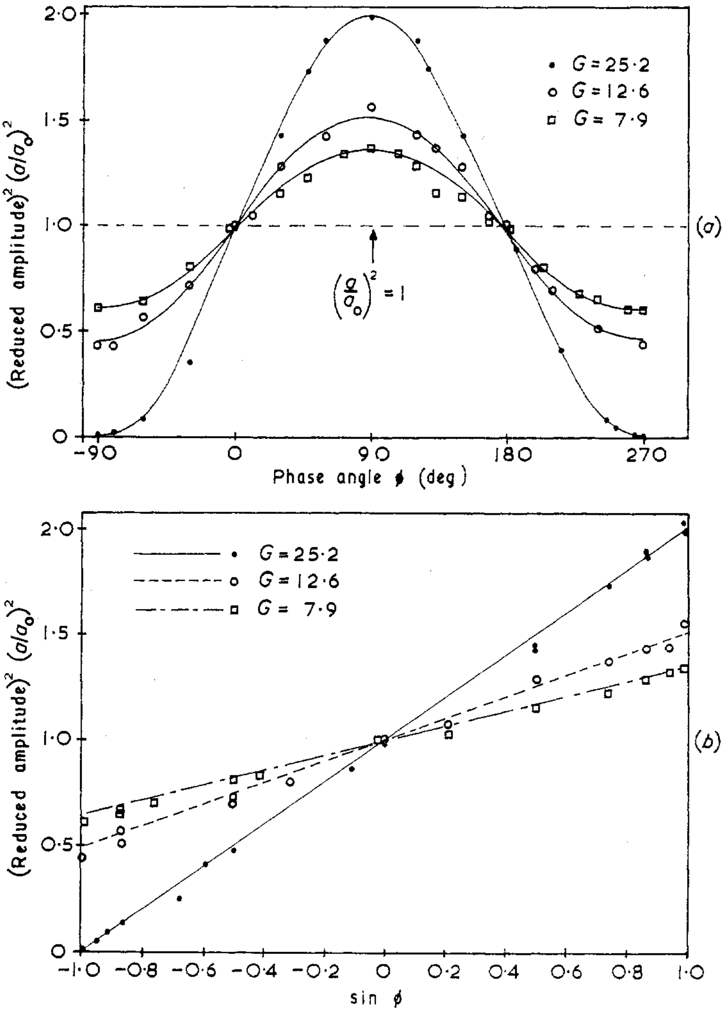


Figure 14. A plot of the (reduced amplitude)$^2(a/a_0)^2$ against (a) phase angle ϕ, and (b) $\sin\phi$.

5.2. *Effect on dc properties of plasma*

In § 4 it was mentioned that, from the change in the density profiles with the instability present and with it suppressed, it was inferred that the cross-field diffusion of plasma had been reduced, but no quantitative measure of this change was ascribed. However, Simon (1955, 1959) has considered the problem of cross-field diffusion in an arc column and has calculated an expression for the density profile $n(r)$ in terms of the cross-field diffusion coefficient D. This is given as

$$n(r) = A\,\mathrm{K}_0(r/r_0) \qquad (21)$$

where K_0 is the Bessel function, A is a constant and r_0 the 'e' folding distance in the plasma. When $r \gg r_0$ this equation (21) reduces to

$$n(r) \simeq (r)^{-1/2} \exp(-r/r_0) \tag{22}$$

and

$$r_0 = \left(\frac{L \, . \, D_{\perp}}{2v_z}\right)^{1/2}$$

where L is the length of the plasma column and v_z the longitudinal ion velocity.

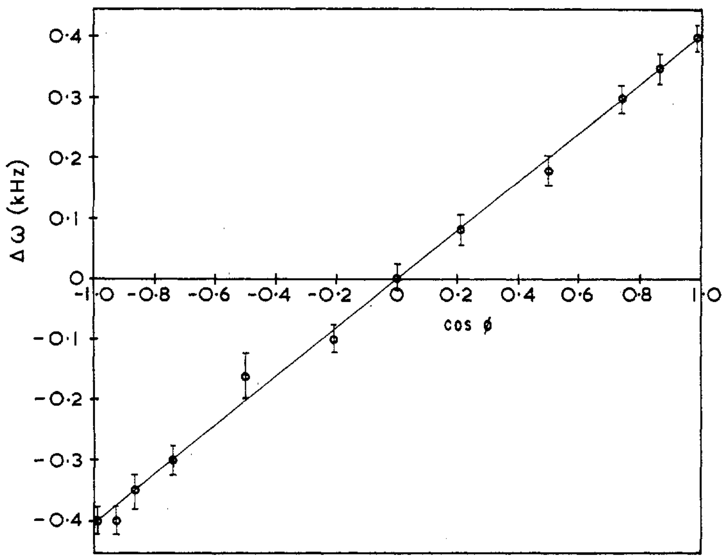


Figure 15. The change in the instability frequency $\Delta\omega$(kHz) plotted against the cosine of the phase angle (cos ϕ).

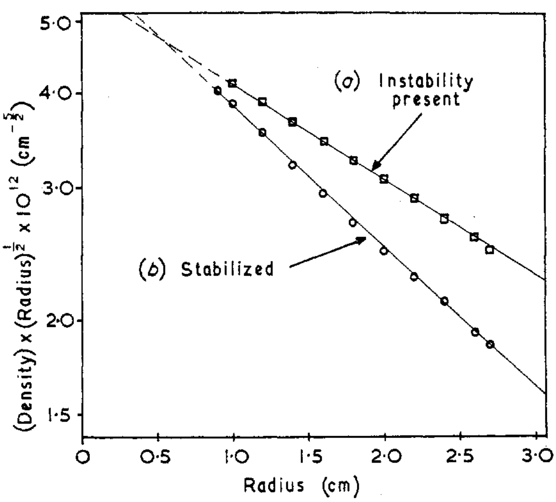


Figure 16. The logarithm of (density) × (radius)$^{1/2}$ plotted against radius for (*a*) instability present, and (*b*) stabilized.

Consequently, if $\ln nr^{1/2}$ is plotted against r, the slope of this curve will give a measure of r_0. This is shown plotted in figure 16 for the instability at 5·5 kHz, under the conditions (*a*) the instability present, and (*b*) the system stabilized. This results in values $r_0(\text{unst})/r_0(\text{stab}) = 1{\cdot}52$. Then, assuming the longitudinal ion velocity v_z is unchanged under the two conditions, a value for the diffusion constant ratio is $D_\perp(\text{unstab})/D_\perp(\text{stab}) = 2{\cdot}3$. In the same way an analysis of the profiles for the instability at 7 kHz when it is suppressed by one- or two-plate systems results in a ratio of $D_\perp(\text{unst}) : D_\perp(\text{stab(1)}) : D_\perp(\text{stab(2)}) = 2{\cdot}6 : 1{\cdot}7 : 1$. In this way it is possible to ascribe some quantitative relative measurement to the effect.

6. Discussions and conclusions

It has been established that a feedback technique with a suitable gain and phase shift value in the feedback loop can be used to suppress a drift instability in a plasma. If the gain is increased above the optimum value g_0 the instability in the system begins to reappear and for large gain values ($g \gg g_0$) the level is larger than its unperturbed amplitude. This is when the loop consisting of the total system (plasma, phase shifter, amplifier, and return to plasma) is caused to oscillate, and then the final amplitude is determined by nonlinear elements in this total system. However, as the gain value is increased from zero to its value g_0, both the linear generalized theory of Taylor (1969) and the phenomenological nonlinear theory developed in this paper give a good description of the experimental data obtained, but give no prediction for gain values greater than g_0. The nonlinear theory assumes that the final limited amplitude can be determined from an equation of the Van der Pol type, and is applicable as long as the growth rate α is much smaller than the instability frequency ω_0. This is satisfied in this case, since $\alpha/\omega_0 \simeq 0{\cdot}1$. This good agreement between experiment and theory is achieved in the case when one suppressor plate is used.

Empirically, it has been found that even in separate azimuthal mode numbers are fed back in the plasma, using up to four separate plates spaced azimuthally in one radial plane around the plasma, best stabilization is achieved with only one suppressor plate. This is when a mixture of modes is fed back into the plasma with the same proportion of frequencies and mode numbers as that present in the instability. Also, it was found experimentally that improved suppression was achieved when two separate feedback systems of pick-up detector, amplifier, phase-shifter and suppressor plate were utilized. Further separate systems were not available to see if increased numbers improved efficiency. From the density profiles measured with and without the feedback system operating, a change in the cross-field diffusion constant $D_\perp$ was inferred. In this particular case, using the theory of Simon (1955, 1959), the diffusion constant $D_\perp$ was found to decrease by a factor of 2·3 when the instability was suppressed.

Therefore, in general, it would appear that this method could be extremely useful to suppress 'drift-type' instabilities in any future thermonuclear containment device. For this application some other method of detection and 'injection' technique, which did not employ probes and plates inside the 'hot' plasma, would be necessary for efficient operation. Among the suggested methods of 'injection' are electric field modulation at the boundary (Arsenin and Chuyanov 1968), neutral particle injection (Chen and Furth 1969), and external magnetic field modulation. Detection of the instability could be achieved by electric field pick up outside the plasma, or emitted radiation detection. Therefore, it would be interesting to perform this type of experiment on a high density, 'hot' plasma, in which external detectors and 'suppressors' were used to suppress internal 'drift-type' instabilities.

Acknowledgments

It is a pleasure to thank N. R. G. Ainsworth and W. H. W. Fletcher for their skilful assistance in performing some of these experiments.

References

ABRAMS, R. H., YADLOWSKY, E. J., and LASHINSKY, H., 1969, *Phys. Rev. Lett.*, **22,** 275.
ALDRIDGE, R. V., and KEEN, B. E., 1970, *Plasma Phys.*, **12,** 1.
ARSENIN, V. V., and CHUYANOV, V. A., 1968, *Atom. Energy* (USSR), **24,** 327 (transl. 1968, *Sov. J. Atom. Energy*, **24,** 407).
ARSENIN, V. V., SHILTSOV, V. A., LIKHTENSHTEIN, V. KH., and CHUYANOV, V. A., 1969, *Sh. Eksperim. Theor. Fiz. Pisma Redakt*, **8,** 69. (transl. 1968, *JETP Lett.*, **8,** 41).
ARTSIMOVICH, L. A., and KARTESHEV, K. B., 1962, *Dokl. Akad. Nauk SSSR*, **146,** 1305 (transl. 1963, *Sov. Phys. Dokl.*, **7,** 919).
BRUNDIN, C. L., 1964, University of California, Berkeley, *Rep.* AS-64-9.
CHEN, F. F., and FURTH, H. P., 1969, Princeton University, *Plasma Phys. Lab. Rep.*, MATT-707.
CHU, T. K., HENDEL, H. W., and POLITZER, P. A., 1967, *Phys. Rev. Lett.*, **19,** 1110.
FURTH, H. P., and RUTHERFORD, P. H., 1969, Princeton University, *Plasma Phys. Lab. Rep.*, MATT-705.
HENDEL, H. W., CHU, T. K., and POLITZER, P. A., 1968, *Phys. Fluids*, **11,** 2426.
JOHNSON, E. O., and MALTER, L., 1950, *Phys. Rev.*, **80,** 58.
KEEN, B. E., and ALDRIDGE, R. V., 1969, *Phys. Rev. Lett.*, **22,** 1358.
KEEN, B. E., and FLETCHER, W. H. W., 1969, *Phys. Rev. Lett.*, **23,** 760.
—— 1970, *Phys. Rev. Lett.*, **24,** 130.
KEMP, R. F., and SELLEN, J. M., 1966, *Rev. sci. Instrum.*, **37,** 455.
LASHINSKY, H., 1965 a, *7th Int. Conf. on Phenomena in Ionized Gases, Belgrade*, Vol. 2, p. 710.
—— 1965 b, *2nd Int. Conf. on Plasma Physics and Controlled Fusion, Culham*, Vol. 1, p. 499.
MINORSKY, N., 1962, *Non-linear Oscillations* (Princeton, N.J.: Van Nostrand).
PARKER, R. R., and THOMASSEN, K. I., 1969, *Phys. Rev. Lett.*, **22,** 1171.
SIMON, A., 1955, *Phys. Rev.*, **98,** 317.
—— 1959, *Introduction to Thermonuclear Research* (New York: Pergamon), p. 165.
SIMONEN, T. C., CHU, T. K., and HENDEL, H. W., 1969, *Phys. Rev. Lett.*, **23,** 568.
STIX, T., 1969, *Phys. Fluids*, **12,** 627.
TAYLOR, J. B., 1969, *Culham Lab. Rep.*, PPN No. 8/69.
VAN DER POL, B., 1922, *Phil. Mag.*, **43,** 700.
WOO, J., and ROSE, D. J., 1967, *Phys. Fluids*, **10,** 893.